The writer would like to point out a number of substantial errors and omissions contained in the paper:

1. Eqs. 2 and 3 on which the "design" Figs. 2–7 are based are *not* correct, since they ignore the inertial coupling between swaying and rocking oscillations of massive foundations. The effect of such coupling may be significant on both translational and rocking displacements and this has been known to the profession for quite some time (7,11,12).

As an example, Fig. 12, taken from Ratay (11), illustrates the possible errors which may arise from ignoring this coupling between swaying and rocking, i.e., from using relations such as Eqs. 2 and 3 of the paper. The figure refers to the response of a massive, rigid, circular foundation having a dimensionless inertia ratio $b' = I/\rho r_0^5$ equal to 1.57—a value which lies within the range of the b' ratios of Figs. 2–7 of the paper. Furthermore, the center of gravity of the foundation lies a distance of 0.5 r_0 from the base, while the foundation mass ratio $b = m/\rho r_0^3$ equals 4.71, and the foundation mass density is 1.50 ρ, in which ρ = the soil mass density and m = the total mass of the foundation. Fig. 12 gives two plots for the normalized amplitude of the angle of rotation experienced by the foundation versus the frequency factor a_0. Curve 1 has been computed rigorously, accounting for the sliding-rocking coupling ("exact" solution). Curve 2 has been derived for "pure" rocking, ignoring the translation of the base of the footing, i.e., by using a one-dimensional expression identical to Eq. 2. It is evident that neither the
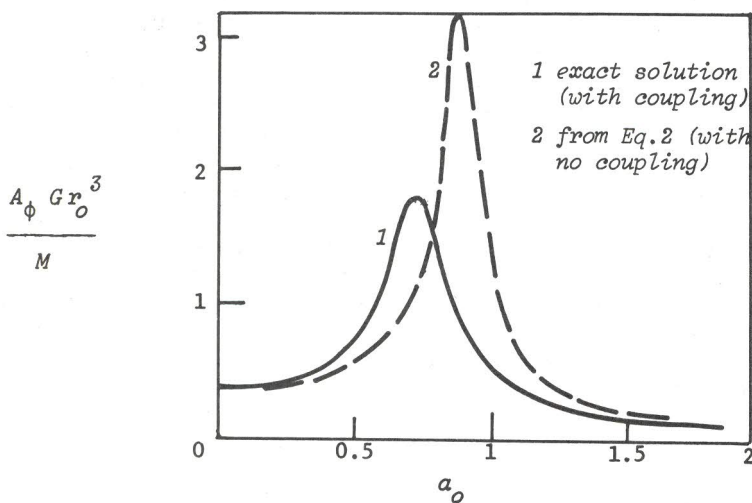

FIG. 12.—Difference in Response Curves Computed from Exact Theory and from Eq. 2 of Paper which Neglects Swaying-Rocking Coupling (11)

[2] Assoc. Prof. of Civ. Engrg., Rensselear Polytechnic Inst., Troy, N.Y. 12181.
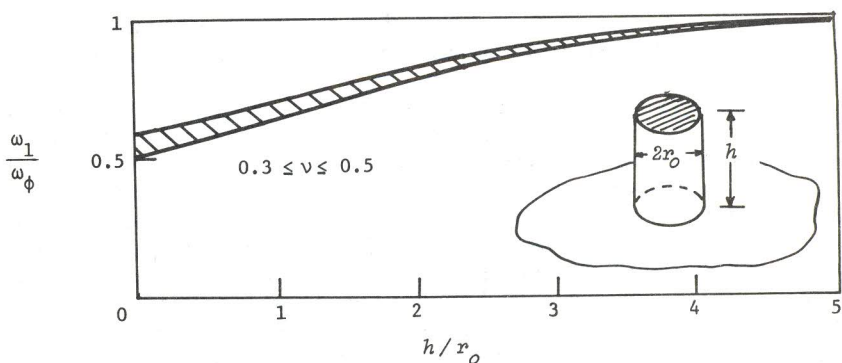
FIG. 13.—Discrepancy between Actual First Resonant Frequency ω_1 and Resonant Frequency ω_ϕ Computed by Neglecting Swaying-Rocking Coupling

resonant frequency nor the resonant peak can be reliably predicted when inertial coupling is ignored.

In general, for a foundation modeled as as a uniform cylinder of radius r_0 and height h, the difference between the actual first resonant frequency of the coupled system, ω_1, and the resonant frequency in "pure" rocking, ω_ϕ, computed from Eq. 2, is a function of h/r_0. Fig. 13 plots the ratio ω_1/ω_ϕ versus h/r_0, for a half space with Poisson's ratio ranging from 0.30–0.50 and a hysteretic damping ratio of 0.03. It is seen that a 50% or more decrease in resonant frequency due to coupling occurs when $h/r_0 < 1$. Only for slender foundations, $h/r_0 > 4$, can ω_ϕ be taken as practically equal to ω_1.

In view of the sensitivity of the dynamic rocking stiffness and damping coefficients k_r and c_r to variations in frequency in the range $0 < a_0 < 2$ [see Fig. 14, taken from Veletsos & Wei (19)], it is not difficult to understand the cause of the erroneous predictions of resonant amplitudes by the "pure" rocking Eq. 2.
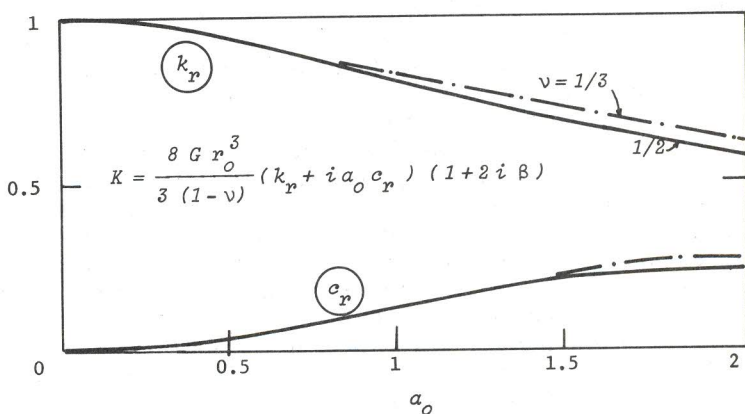

FIG. 14.—Variation of Stiffness and Damping Coefficients k_r and c_r with Frequency Factor a_0 (19)

In conclusion, the graphs presented in the paper are only approximately correct for relatively slender foundations, i.e., for high inertia ratios b'.

2. The curves shown in Fig. 2 seem to be inconsistent with those in Figs. 4 and 6. How is it possible to have very similar resonant amplitudes of rotation from a "uniform" and a "parabolic" normal stress distributions, which are drastically different with reference to rocking, while the "rigid-base" peak rotations are only about 1/2 or less of the "uniform" rotations despite the relative similarity of the corresponding two stress distributions? The writer suspects that, if not due to a computational error, the discrepancies between Fig. 2 and Figs. 4 and 6 may be the result of the author's use of the locally *maximum* angle $\phi = \phi(r, \theta)$ (instead of the appropriate weighted average) for the "uniform" and "parabolic" distributions. Reference is made to Housner & Castellani (21) for a pertinent discussion regarding the effect of contact stress distributions on the vertical vibrations of massive foundations. A comment by the author on this subject will be appreciated.

3. The horizontal axis passing through the center of the soil-footing interface is referred to as "*the* axis of rotation" (pages 907, 909). Thus, in order to estimate rotation amplitudes from the field tests the author first calculates "the amplitude corresponding exactly at the bottom surface of the footing, which represents the amplitude of sliding motion, A_x" (page 913).

In fact, the exact location of the axis of rotation depends on the frequency of vibration and only by shear coincidence will it be located at the foundation base. If for a given frequency factor a_0, $A_x = A_x(a_0)$ and $A_\phi = A_\phi(a_0)$ represent the amplitudes of base translation and foundation rotation, respectively, the axis of rotation will be located a distance $z = z(a_0) = -(A_x/A_\phi) \cdot \exp(i\phi)$ from the base (positive z is measured upward and ϕ is the phase shift between x and ϕ). At relatively small frequencies z is negative and the axis of rotation lies below the base; the opposite is true at higher frequencies.

In any case, it is important to understand that both A_x and A_ϕ are influenced by rocking as well as swaying, due to the aforementioned inertial coupling. The author's claim that A_x represents the amplitude of swaying alone, is equivalent to assuming an artificial uncoupling of the two vibrational modes.

4. Finally, it is noted that in the paper no mention is made of material damping in the soil. Yet, for rocking oscillations, this type of damping (of a hysteretic nature) is quite important due to the very small radiation damping, especially at low frequency factors (or high inertia ratios, b'). Accounting for even a 2 or 3% material damping may reduce the computed resonant peaks by at least 50% in many cases. The counter argument that is often heard, i.e., that ignoring damping leads to errors "on the safe side," is unacceptable to the writer in view of the substantial progress during the last two decades in the state-of-the-art of analyzing foundation vibrations (22) and of estimating dynamic soil properties in the field and laboratory (23).

Appendix.—References

21. Housner, G. W., and Castellani, A., Discussion of the paper "Comparison

of Footing Vibration Tests with Theory" by F. E. Richart and R. V. Whitman, *Journal of the Soil Mechanics and Foundation Division*, ASCE, Vol. 95, 1969, pp. 360–364.

22. Gazetas, G., "Analysis of Machine Foundation Vibrations: State of the Art," *International Journal of Soil Dynamics and Earthquake Engineering*, Vol. 2, No. 1, 1983.

23. Woods, R. D., "Measurement of Dynamic Soil Properties," *Earthquake Engineering and Soil Dynamics*, ASCE, Vol. 1, 1978, pp. 91–178.